

# On creation of scalar particles with Gauss-Bonnet type coupling to curvature in Friedmann cosmological models<sup>1</sup>

Yu. V. Pavlov\*

*Institute of Problems in Mechanical Engineering, Russian Acad. Sci.,  
Bol'shoy pr. 61, St. Petersburg 199178, Russia;*

*A. Friedmann Laboratory for Theoretical Physics, St. Petersburg, Russia;*

*Copernicus Center for Interdisciplinary Studies, Kraków, Poland*

**Abstract.** Calculations are presented for creation of massive and massless scalar particles coupled to Gauss-Bonnet type curvature in Friedmann cosmological models. It has been shown that, for fields of mass  $m$ , the effect of the coupling parameter  $\zeta$  with the Gauss-Bonnet invariant is insignificant if  $\zeta m^2 \ll 1$ . In all cases under consideration, the created particle number is compatible by order of magnitude with the number of causally disconnected space-time regions by the Compton time, corresponding  $1/m$  or  $\sqrt{\zeta}$ .

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## 1. Introduction

Quantum effects in the gravitational field have been actively studied since the 70s of the last century. Quantum field effects, in particular, particle creation can have important applications in the early Universe cosmology and astrophysics [1, 2]. A basic contribution to this branch of theoretical physics has been made by A.A. Grib and his students. In [3], A.A. Grib and S.G. Mamayev have suggested the Hamiltonian diagonalization method to describe particle creation by the gravitational field. For the first time, they obtained finite expressions for the particle number density created in homogeneous and isotropic models of the Universe. A great significance of the approach suggested in [3] is connected with the definitions of the notions of a vacuum and an elementary particle in curved space-time, where, in general, there is no symmetry group like the Poincaré group in Minkowski space. If one assumes that a particle is associated with a quantum of energy, then observation of a particle at a certain time instant means, according to the principles of quantum theory, finding an eigenstate of the Hamiltonian. That is what is taken into account in the Hamiltonian diagonalization method [1].

Numerical estimates have shown that the effect of particle creation with masses of the order of the proton mass is negligibly small in cosmology. However, if the particle masses are large, e.g., those of the order of the Grand Unification scale, then the number of particles created in the Friedmann Universe will be of the order of the Eddington-Dirac number ( $\approx 10^{80}$ ) [4, 5]. This can be used in order to explain the observed baryonic charge of the Universe and the origin of ultra-high energy cosmic rays [6]–[8].

Up to a certain time, the particle creation theory considered only the case of conformally coupled fields. For fields with more general couplings with the curvature it was shown, e.g., in [9, 10], that the corresponding nonconformal additions can be dominant in both the particle creation effect and in the vacuum mean values of the stress-energy tensor components. For nonconformally coupled fields, the metric Hamiltonian diagonalization method led to infinite values of the created particle number [11]. This problem has been solved in [12, 13]. Calculations of nonconformal particle creation with minimal coupling to the curvature in Friedmann universes are presented in [14].

In the present paper we consider the case of nonconformal coupling with the Gauss-Bonnet type curvature. This type of scalar field coupling to gravity is a generalization of the usual nonconformal coupling that does not change the kinetic term and

\*yuri.pavlov@mail.ru

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preserves the most important property of a scalar field: its metric stress-energy tensor in this case also does not contain higher than second-order derivatives of the metric and the field itself. For a model metric, admitting an exact analytical solution, we have previously shown [15] that the existence of a coupling with the GaussBonnet invariant can exert a dominant influence in the particle creation effect by the gravitational field. In the present paper we study particle creation in Friedmann cosmological models for particles coupled with the Gauss-Bonnet type curvature.

We use the system of units in which  $\hbar = c = 1$ . The signs of the Riemann and Ricci tensors are chosen so that  $R^i_{jkl} = \partial_l \Gamma^i_{jk} - \partial_k \Gamma^i_{jl} + \Gamma^i_{nl} \Gamma^n_{jk} - \Gamma^i_{nk} \Gamma^n_{jl}$ ,  $R_{ik} = R^l_{ilk}$ , where  $\Gamma^i_{jk}$  are the Christoffel symbols.

## 2. Scalar field with Gauss-Bonnet type coupling

Consider a complex scalar field  $\varphi(x)$  of mass  $m$  with the Lagrangian

$$L(x) = \sqrt{|g|} [g^{ik} \partial_i \varphi^* \partial_k \varphi - (m^2 + \xi R + \zeta R_{GB}^2) \varphi^* \varphi] \quad (1)$$

and the corresponding equation of motion

$$(\nabla^i \nabla_i + m^2 + \xi R + \zeta R_{GB}^2) \varphi(x) = 0, \quad (2)$$

where  $\nabla_i$  are covariant derivatives in  $N$ -dimensional space-time with the metric  $g_{ik}$ ,  $g = \det(g_{ik})$ ,  $R$  is the scalar curvature,

$$R_{GB}^2 \stackrel{\text{def}}{=} R_{lmpq} R^{lmpq} - 4R_{lm} R^{lm} + R^2, \quad (3)$$

$\xi$  is a dimensionless constant while the constant  $\zeta$  has the dimension of the inverse mass squared:  $\zeta = 1/M_\zeta^2$ . The integrals of  $R$  and  $R_{GB}^2$  are proportional, in the dimensions  $N = 2$  and  $4$ , respectively, to the Euler characteristics of the manifolds, which are, according to the Gauss-Bonnet theorem, topological invariants. Therefore the coupling to curvature of the form (1), is called, at  $\zeta \neq 0$ , the Gauss-Bonnet type coupling.

Eq. (2) is conformally invariant if  $m = 0$ ,  $\zeta = 0$  and

$$\xi = \xi_c \stackrel{\text{def}}{=} \frac{N-2}{4(N-1)} \quad (4)$$

( $\xi_c = 1/6$  for  $N = 4$ ). Therefore the case  $\xi = \xi_c$ ,  $\zeta = 0$  is called conformal coupling to the curvature. The case  $\xi = 0 = \zeta$  corresponds to minimal coupling.

For the GaussBonnet type coupling, the metric stress-energy tensor (SET) of a scalar field (1) does

not contain derivatives of the metric higher than second-order. Explicit expressions for the SET are given in [16], where, for the first time, quantization and renormalization of the vacuum mean values of SET components were carried out for a scalar field with this type of coupling in a homogeneous and isotropic space-time. Taking into account the possible scalar field coupling with the Gauss-Bonnet invariant  $R_{GB}^2$  can be significant for the early Universe, in black holes studies, in calculations of boson stars parameters, etc.

We write the metric of a homogeneous and isotropic space-time in the form

$$ds^2 = dt^2 - a^2(t) dl^2 = a^2(\eta) (d\eta^2 - dl^2), \quad (5)$$

where  $dl^2$  is the metric of an  $(N-1)$ -dimensional space of constant curvature  $K = 0, \pm 1$ . In the metric (5), the expressions for the scalar curvature and for the GaussBonnet invariant are (see [16])

$$R = a^{-2}(N-1) [2c' + (N-2)(c^2 + K)], \quad (6)$$

$$R_{GB}^2 = a^{-4}(N-1)(N-2)(N-3)(c^2 + K) \times [4c' + (N-4)(c^2 + K)], \quad (7)$$

where the prime denotes a derivative in the conformal time  $\eta$ ,  $c = a'/a = \dot{a}(t)$ .

The Einstein equations

$$R_{ik} - \frac{1}{2}g_{ik}R = -8\pi G T_{ik}, \quad (8)$$

for the metric (5) have the form

$$\frac{c^2 + K}{a^2} = \frac{16\pi G \varepsilon}{(N-1)(N-2)}, \quad (9)$$

$$-\frac{1}{a^2} \left[ c' + \frac{N-3}{2}(c^2 + K) \right] = \frac{8\pi G p}{N-2}, \quad (10)$$

where  $\varepsilon$  and  $p$  are the energy density and pressure of the background matter whose SET is

$$T_k^i = \text{diag}(\varepsilon, -p, \dots, -p). \quad (11)$$

For spatially flat cosmologies ( $K = 0$ ), in the case where the pressure of the background matter is proportional to its energy density,

$$p = w\varepsilon, \quad w = \text{const}, \quad (12)$$

at  $w > -(N-3)/(N-1)$  from (9), (10) we obtain

$$a = a_0 t^q = a_1 \eta^\beta, \quad (13)$$

where  $t, \eta \in (0, \infty)$ ,

$$q = \frac{2}{(N-1)(w+1)}, \quad \beta = \frac{q}{1-q}, \quad (14)$$

$$t = \frac{a_1}{\beta+1} \eta^{\beta+1}, \quad a_0 = a_1^{1/(\beta+1)} (\beta+1)^{\beta/(\beta+1)}. \quad (15)$$

In the range under consideration

$$w \in \left( -\frac{N-3}{N-1}, +\infty \right) \quad (16)$$

the exponents in the scale factor change in the intervals  $q \in (0, 1)$ ,  $\beta \in (0, +\infty)$ . The values

$$w = 0, \quad q = \frac{2}{N-1}, \quad \beta = \frac{2}{N-3} \quad (17)$$

correspond to dust background matter, and

$$w = \frac{1}{N-1}, \quad q = \frac{2}{N}, \quad \beta = \frac{2}{N-2} \quad (18)$$

to a radiation-dominated epoch.

### 3. Scalar particle creation in a homogeneous isotropic space

To calculate the number of created particle pairs we use the Hamiltonian diagonalization method [1]. The full set of solutions to Eq. (2) in the metric (5) can be found in the form

$$\varphi(x) = a^{-(N-2)/2}(\eta) g_\lambda(\eta) \Phi_J(\mathbf{x}), \quad (19)$$

where

$$g_\lambda''(\eta) + \Omega^2(\eta) g_\lambda(\eta) = 0, \quad (20)$$

$$\Omega^2(\eta) = m^2 a^2 + \lambda^2 + (\xi - \xi_c) a^2 R + \zeta a^2 R_{GB}^2, \quad (21)$$

$$\Delta_{N-1} \Phi_J(\mathbf{x}) = -\left( \lambda^2 - \left( \frac{N-2}{2} \right)^2 K \right) \Phi_J(\mathbf{x}), \quad (22)$$

$J$  being the set of indices (quantum numbers), numbering the eigenfunctions of the Laplace-Beltrami operator  $\Delta_{N-1}$  in  $(N-1)$ -dimensional space.

According to the Hamiltonian diagonalization method, the functions  $g_\lambda(\eta)$  should satisfy the following initial conditions [12, 13]:

$$g_\lambda'(\eta_0) = i \Omega(\eta_0) g_\lambda(\eta_0), \quad |g_\lambda(\eta_0)| = \Omega^{-1/2}(\eta_0). \quad (23)$$

If a quantum scalar field is in a vacuum state at the time instant  $\eta_0$ , then the density of particle pairs created by the time  $\eta$  can be calculated (for the metric with  $K = 0$ ) by the formula [1]

$$n(\eta) = \frac{B_N}{2a^{N-1}} \int_0^\infty S_\lambda(\eta) \lambda^{N-2} d\lambda, \quad (24)$$

where  $B_N = [2^{N-3} \pi^{(N-1)/2} \Gamma((N-1)/2)]^{-1}$ ,  $\Gamma(z)$  is the gamma function,

$$S_\lambda(\eta) = \frac{|g_\lambda'(\eta) - i \Omega g_\lambda(\eta)|^2}{4\Omega}. \quad (25)$$

As shown in [12], [13],  $S_\lambda \sim \lambda^{-6}$  and the integral in (24) converges at  $N < 7$ .

The formula for the number of particle pairs created in the volume  $a^{N-1}(t)$  by the time  $t$ , in the case of the scale factor (13), can be written in the form

$$N(t) = \left( \frac{a(t_*)}{t_*} \right)^{N-1} b_q^{(0)}(t), \quad (26)$$

where  $t_*$  is a certain fixed time instant. The superscript (0) indicates that it is scalar particle creation that is considered. As follows from (26),  $b_q^{(0)}(t)/(1-q)^{N-1}$  is the proportionality factor between the created particle number and the number of causally disconnected regions at the time  $t_*$  after the Big Bang.

Some explanations are in order. Suppose that at the Big Bang time, from each spatial point, some signals have been emitted, propagating with the speed of light. Let us find the distance  $D_H(t)$  (the cosmological horizon size) to which these signals come from the emission point by the instant at which the age of the Universe is equal to  $t$ . The null geodesic equation  $ds = 0$  in the metric (5) reduces to  $dt = \pm a(t) dl$ , therefore,

$$D_H(t) = a(t) \Delta l = a(t) \int_0^t \frac{d\tau}{a(\tau)}. \quad (27)$$

For the power-law scale factor (13)

$$D_H(t) = t/(1-q). \quad (28)$$

The number of causally disconnected regions in the volume  $a^{N-1}(t)$  is equal to

$$N_c = \left( \frac{a(t)}{D_H(t)} \right)^{N-1} = (1-q)^{N-1} \left( \frac{a(t)}{t} \right)^{N-1}, \quad (29)$$

which confirms the above interpretation of Eq. (26).

In the case of the usually considered coupling between the scalar field and the curvature,  $\xi R \varphi^* \varphi$ , one uses as the time  $t_*$  in Eq. (26) the Compton time  $t_C = 1/m$  for a massive scalar field. Let us note that for a conformally coupled scalar field the results of calculations of the created particle number do not depend on the instant when the initial conditions are imposed as well as on the instant when the

created particles are being observed if these times are much smaller and much larger than the Compton time, respectively. Thus the coefficient  $b_q^{(0)}(t)$  for a conformally coupled field is time-independent at  $mt \gg 1$ .

The relationship between the created particle number and the number of causally disconnected regions in Friedmann cosmologies at Compton time has been pointed out by Grib (see, e.g., [4]). The results of numerical calculations of the coefficient  $b_q^{(0)}$  in four-dimensional space-time for power-law scale factors are presented, e.g., in [14] (see also [17]).

For a nonconformal scalar field, the results can crucially depend on the choice of the initial time instant because  $\Omega^2(\eta)$  can grow without limits at fixed  $\lambda$  due to the growth of  $|R(t)|$  and  $|R_{GB}^2(t)|$  as  $t \rightarrow 0$  (see (6), (7), (21)). However, under the condition

$$(\xi - \xi_c)R(t) + \zeta R_{GB}^2(t) < 0, \quad t \rightarrow 0, \quad (30)$$

the initial time  $t_0$  in the particle creation problem is determined by the requirement  $\Omega^2(t) \geq 0$  for  $t > t_0$  and any momenta  $\lambda$ . In the opposite case ( $\Omega^2(t) < 0$  for some values of  $\lambda$ ), in the presence of a scalar field self-interaction, the vacuum state would be reconstructed similarly to the symmetry violation mechanism. Thus we choose the initial time  $t_0$  from the equality

$$m^2 + (\xi - \xi_c)R(t_0) + \zeta R_{GB}^2(t_0) = 0. \quad (31)$$

For a nonconformal scalar field such an approach to the particle creation problem was suggested for the first time in [14], where the calculation results are presented for a scalar field with minimal coupling and power-law scale factors.

In the present work, particle creation has been studied for a 4D homogeneous and isotropic space-time with flat spatial sections ( $K = 0$ ). For the power-law scale factors (13), exact solutions to Eq. (20) with GaussBonnet type coupling are unknown. Therefore, numerical calculations have been conducted for such particle creation, and their results are presented in the following plots.

Let us note that in the radiation-dominated case ( $a = a_0\sqrt{t}$ ), presented in Fig. 1, the scalar curvature  $R = 0$  and therefore the value of the parameter  $\xi$  does not affect particle creation. The time  $t_0$ , according to (31), is

$$t_0 = \sqrt[4]{\frac{3\zeta}{2m^2}}. \quad (32)$$

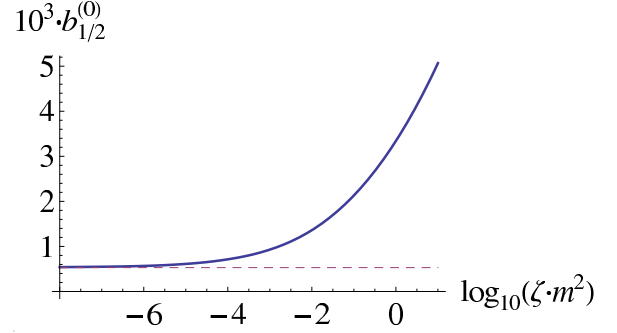


Figure 1: Particle creation in a radiation-dominated Universe ( $w = 1/3$ ).

The dashed line in Fig. 1 corresponds to the value  $b_{1/2}^{(0)} = 5.3 \cdot 10^{-4}$  for a conformally coupled scalar field in a radiation-dominated Universe [18].

In the case of a dust-dominated Universe,  $p = 0$  (the scale factor  $a = a_0 t^{2/3}$ ), the scalar curvature  $R \neq 0$ , and the value of  $\xi$  affects particle creation, as is evident in Fig. 2.

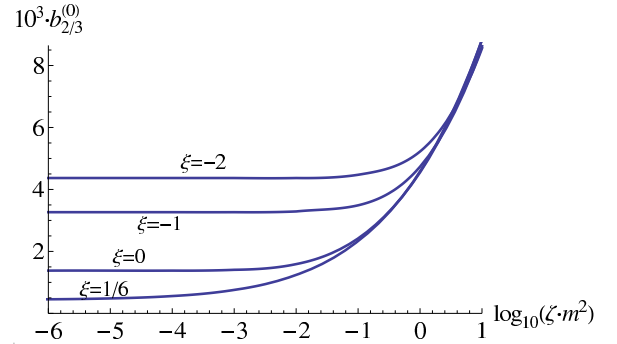


Figure 2: Particle creation in a dust-dominated Universe ( $p = 0$ ).

In both Figs. 1 and 2, the parameter  $b_q^{(0)}$  was determined relative to the time  $t_* = 1/m$  (see Eq. (26)). The value of  $b_q^{(0)}$  was practically independent of the specific choice of the final instant  $t$  in the case  $t \gg 1/m$ .

As is evident from Figs. 1 and 2, the influence of the coupling parameter  $\zeta$  with the GaussBonnet invariant on particle creation is insignificant if  $\zeta \ll 1/m^2$ . The number of created particles in both cases, at  $\zeta \leq 1/m^2$ , is comparable by order of magnitude with the number of causally disconnected regions in the corresponding Friedmann model by the Compton time ( $t_C = 1/m$ ) for the scalar field mass  $m$  from the beginning of the expansion.

For a massless scalar field, the time  $t_*$  in Eq. (26) was chosen as  $t_* = \sqrt{\zeta}$ , i.e., the Compton time

for the mass scale of the coupling parameter  $\zeta$  between the scalar field and the Gauss-Bonnet invariant  $R_{GB}^2$ . In the massless case, the results of calculations of particle creation for different values of the exponent  $q$  of the scale factor (13) are presented in Fig. 3.

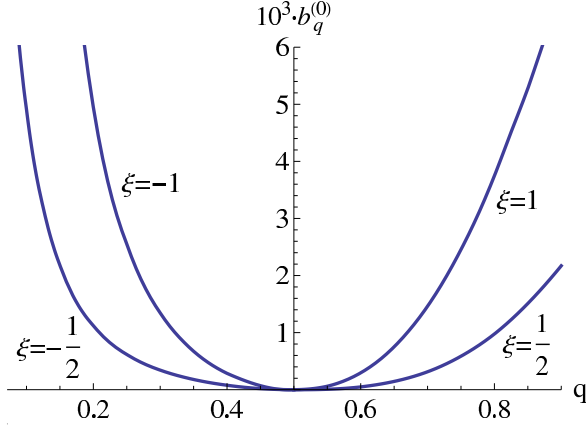


Figure 3: Massless scalar particle creation for power-law scale factors.

To provide the conditions (30), (31), it was assumed

$$\zeta > 0, \quad (\xi - \xi_c)(2q - 1) > 0, \quad (33)$$

$$t_0 = 2q \sqrt{\frac{\zeta(1-q)}{(\xi - \xi_c)(2q - 1)}}. \quad (34)$$

The whole  $\zeta$  dependence of the created particle number is determined in this case by the factor  $(a(t_*)/t_*)^3$  in Eq. (26) and is proportional to  $\zeta^{-3(1-q)}$  (see Eq. (29)). The decrease in the particle number with growing  $\zeta$  is explained in this case by the growth of the time value  $t_0$ , at which the initial vacuum state is postulated. As time grows, the gravitational field becomes weaker and creates a smaller number of particles.

In all cases considered (see Figs. 1–3), the coefficient  $b_q^{(0)}$  has the order  $\approx 10^{-4} - 10^{-2}$ . Therefore, as in the case of conformally coupled particles in Friedmann models, the number of particles created by the gravitational field of the expanding Universe is comparable with the number of causally disconnected regions by the Compton time  $t_C$  from the expansion beginning. For a massive field, at  $\zeta m^2 \leq 1$ , this time is determined by the field mass,  $t_C = 1/m$ . For a massless field, this time corresponds to the mass scale of the Gauss-Bonnet coupling parameter  $t_C = \sqrt{\zeta}$ .

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